Enrollment No: _______________________ **Exam Seat No:**_______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name : Graph Theory

Subject Code : 5SC03GTE1 **Branch :** M.Sc (Mathematics)

Semester : III **Date:** 08/12/2015 **Time :** 2:30 **To** 5:30 **Marks :** 70

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 Attempt the Following questions.

a. Give an example of graph which has a Hamiltonian circuit but not an Euler circuit. **(02)**

b. Draw the undirected graph represented by incidence matrix given below. **(02)**

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

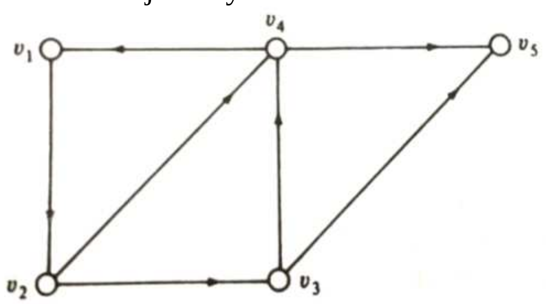
c. How many edges are there in undirected graph with 6 vertices each of degree 5? **(02)**

d. Define Euler graph. **(01)**

Q-2 Attempt all questions

A From the digraph given below, answer the following: **(07)**

1. Find in degree, out degree and total degree of each vertex.
2. Find reachable set of each vertex.
3. Find all node bases.
4. Find all strong components.
5. Write the adjacency matrix.



B Prove that all paths are elementary in a tree. **(07)**

OR

Q-2 Attempt all questions

A Give three different representations of a tree from **(07)**

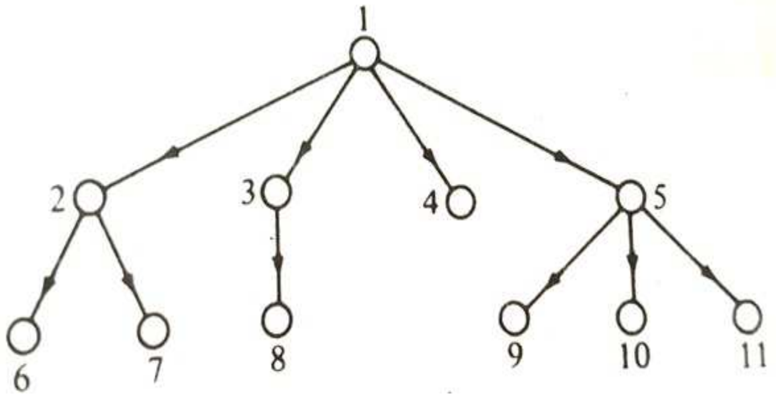
$$\left(v_0\left(v_1\left(v_2\right)\left(v_3\left(v_4\right)\left(v_5\right)\right)\right)\left(v_6\left(v_7\left(v_8\right)\right)\left(v_9\right)\left(v_{10}\right)\right)\right)$$

Also identify root, branch nodes and leaf nodes from the tree.

B Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even degree. **(07)**

Q-3 Attempt all questions

A Obtain binary tree equivalent to the tree given below: **(07)**

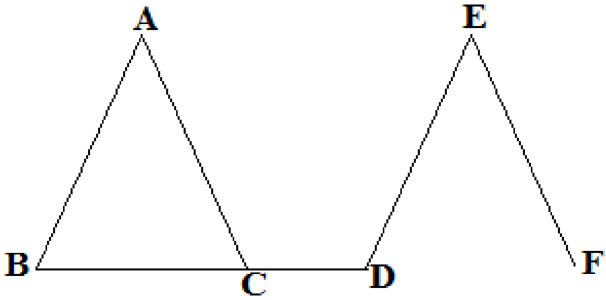


B Prove that an arborescence is a tree in which every vertex other then the root has an in degree of exactly one. **(07)**

OR

Q-3 Attempt all questions

A Define Spanning tree and draw all possible spanning trees of the graph given below: **(07)**

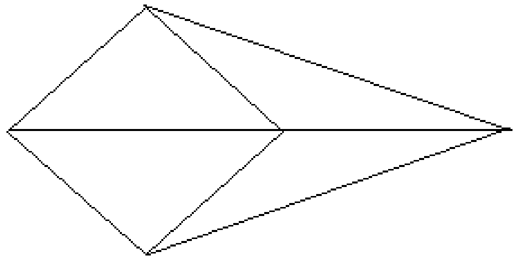


B Prove that in a complete graph with n vertices there are $\frac{n-1}{2}$ edge – disjoint Hamiltonian circuits if n is an odd number ≥ 3. **(07)**

SECTION – II

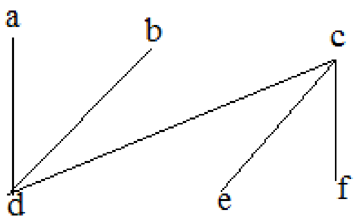
Q-4 Attempt the Following questions.

a. Define proper coloring and determine chromatic number of the graph given below. **(02)**



b. State Hall's matching condition. **(02)**

c. For the bipartite graph given below, find an independence number. **(02)**

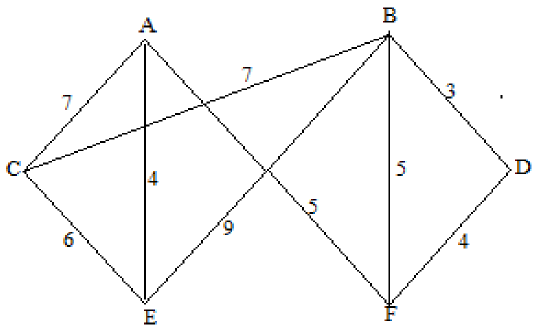


d. Define: Edge cover. **(01)**

Q-5 Attempt all questions

A Prove that every tree with two or more vertices is 2 – chromatic. **(07)**

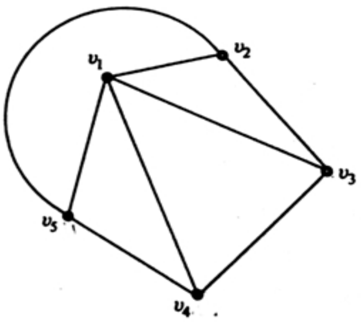
B Define minimum spanning tree and determine minimum spanning tree for the graph given below. **(07)**



OR

Q-5 Attempt all questions

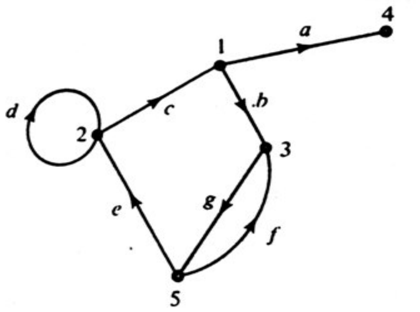
B Define chromatic polynomial and find the chromatic polynomial of the graph given below: **(07)**



B Prove that for every k – regular bipartite graph has a perfect matching. Where k > 0. **(07)**

Q-6 Attempt all questions

A Define adjacency matrix of a digraph and obtain adjacency matrix for the graph given below: **(07)**



B Prove that an n-vertex graph is a tree if and only if its chromatic polynomial **(07)**

$P_n(\lambda) = \lambda(\lambda - 1)^n$

OR

Q-6 Attempt all Questions

A For a bipartite graph given below, Answer the following: **(07)**

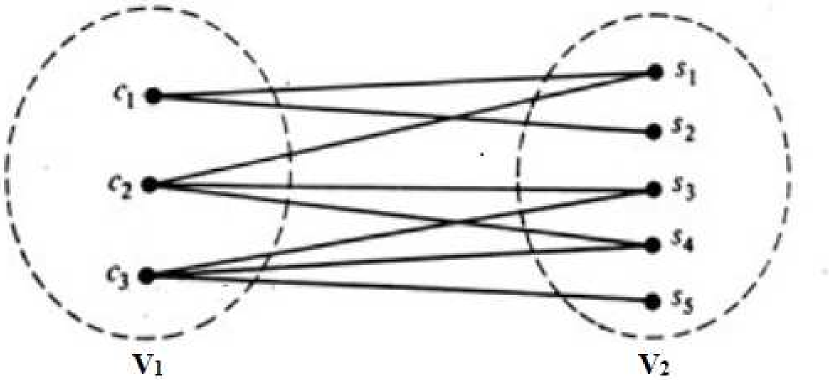
(i) Find complete matching of set V_1 into V_2 .

(ii) Find deficiency.

(iii) Determine maximum number of vertices in set V_1 that can be matched into V_2 .

(iv) Find adjacency matrix of a graph.

(v) Represent complete matching of V_1 into V_2 in matrix form.



B If G is a bipartite graph then prove that the maximum size of a matching in G equals the minimum size of a vertex cover of G. **(07)**